Astr 118, Physics of Planetary Systems
Discussion Week 3: Orbits
Aditya Sengupta, adityars@ucsc.edu, ISB 127

1. The stable orbit of a light object (like a planet) around a heavy object (like a star) is described by an ellipse. We'll use Desmos and the JPL orbit visualizer to look at the shapes of these orbits and how they relate to physical quantities.
   a. Make an ellipse on Desmos, and put the semi-major axis *a* and eccentricity *q* on sliders. (We should use *e*, but Desmos reserves *e* for Euler's constant.) What's the allowed range of *q*, and what happens to the ellipse's shape if you go outside that range?
   b. Add a point representing the star (one of the foci) that moves with *a* and *q*.
   c. Energy (in a dimensionless form that drops some constants and assumes star mass >> planet mass) is related to eccentricity by $q = \sqrt{1 + EL^2/m}$ where *m* is the smaller body's mass. Add sliders for *m, L,* and *E*. What happens to the orbit's shape as you vary all of these? What happens to the star's location?
   d. *Circularization* is a process in which tidal forces make an orbit more circular. What happens to the energy of the orbit as it gets circularized? What happens to the angular momentum?
   e. Find and plot an equation for the curve tracing the center of mass. What can we say about the center of mass motion in general (when we don't have star mass >> planet mass)?
2. Open the JPL orbit visualization tool, ssd.jpl.nasa.gov/tools/orbit_diagram.html.
   a. Mercury's perihelion distance is 0.31 au. Keeping this the same, how eccentric would its orbit have to be to intersect with Earth's? How would you do this without the visualization tool? (Set other parameters to 0.)
   b. How does the orbital period change with eccentricity and perihelion distance? Try eccentricity = 0 first to get a scaling relation, then try again with some eccentricity. How does this relate to Kepler's third law?
   c. Based on your answers to b and to 1c, how does angular momentum affect orbital period at constant eccentricity?
   d. (Bonus) In total, there are six parameters determining an orbit, because we need to set three position and three velocity components. We don't need to know the math behind all of them, but if you have time, vary the other ones! What do they change about the orbit's shape?
3. Our solar system's Planet Nine, if it exists, has a semi-major axis of about 500 au. Roughly what would its orbital period be? How would your answer change if the Sun had 4 times its actual mass?