Astr 118, Physics of Planetary Systems
Discussion Week 6: Formation

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1 Centrifugal and gravitational balance

The material in disks around stars does not undergo gravitational freefall because of the centrifugal force. Suppose a test particle on a wide (circular) orbit moved radially inwards while maintaining a constant angular momentum. How does the centrifugal force vary with r? Find the critical radius at which the two forces balance.

2 Scale heights

We often describe the density of something that falls off exponentially in terms of a *scale height*: the vertical distance over which the density decreases by a factor of e. This can be encapsulated in an expression like $\rho(z) = \rho(0)\exp(-z/H)$.

a. The scale height of Earth's atmosphere is about 10km. Where we are now (Social Sciences 2/generally around the highest point of UCSC), we're about 250m above sea level. What is the air pressure up here relative to sea level? How about at Lick Observatory, which is about 1300m above sea level?

b. The densities of disks drop off as $\rho(z) = \rho(0)\exp(-z^2/2H^2)$. Justify why we can approximate this decently well as $\rho(z) = \begin{cases} \rho(0) & |z| \leq H \\ 0 & |z| > H \end{cases}$. Use whatever method you find easiest (sketching, hand calculations, Python, Desmos, WolframAlpha, etc.)

3 How long does a planet take to form?

Let's estimate a mass accretion rate, and from there estimate how long it'll take to form a planet. The mass accretion rate we're looking for is $\dot{M} = \rho\sigma v$: the density of particles in the disk, times the cross-sectional area we're collecting onto, times the velocity of a typical particle.

a. **Find $\boldsymbol{\rho}$**: The surface density of a disk is about $\Sigma(r) = 2000(r/\text{ au})^{-3/2}\text{g/cm}^2$ (amount of mass per unit area), about 1% of which is solid particles that can coagulate to form a planet. Write down an expression for $\rho(r)$, using the scale height as a variable $H(r)$.

b. **Find $\boldsymbol{\sigma}$**: Write down an expression for σ for a planet of mass M and internal density about 1g/cm^3. (This is varying throughout the accretion process, but we're working to order of magnitude so we'll just use the final value.)

c. **Find $\boldsymbol{v}$**: We're interested in the *relative* velocity between two particles: they're all co-orbiting, so their orbit doesn't induce collisional motion. Instead, we'll say that on average a particle traces out a circle of radius $H(r)$ in the vertical (out-of-disk) direction every period. Find the out-of-disk particle velocity, which we'll say is about the relative velocity we're interested in. You can assume the star is the Sun, so $(P/\text{yr})^2 = (r/\text{au})^3$.

d. Multiply your answers to a, b, c together to get $\dot{M}(r)$.

e. Estimate a timescale for formation via accretion as a function of separation: $t_{\text{accr}} = M_{\text{planet}}/\dot{M}(r)$. You should have M_{planet} and r as parameters.

f. What is this timescale for Earth and for Neptune?