Accretion timescale calculation

We want to find $\dot{M} = \rho\sigma v$.

- $\rho = \Sigma/2H(r) = 0.01 \times 2000\text{g/cm}^2 \cdot (r/\text{au})^{-3/2}/2H(r) = 10\text{g/cm}^2 \cdot (r/\text{au})^{-3/2}/H(r)$. $H(r)$ cancels later on so I'll leave it like this for now.
- $\sigma = \pi R^2 = \pi\left(\frac{3M_\text{planet}}{4\pi\rho_\text{internal}}\right)^{2/3}$; we take the cross-sectional area in terms of the planet radius and convert it so it's in terms of the planet mass and density.
- $v = 2\pi H(r)/P$. I'll say we're working with our solar system so $P/\text{year} = (r/\text{au})^{3/2}$, and our factor of $1/P$ will end up as $1/P = \left(\frac{r}{\text{au}}\right)^{-3/2}\text{year}^{-1}$. So we have $v = 2\pi H(r) \times (r/\text{ au})^{-3/2}\text{year}^{-1}$. (This is a velocity because we have a distance term in $H(r)$, divided by a time from the year^{-1}.)

Multiplying these together, cancelling the $H(r)$s, and combining the (r/au)s, we get

$$\dot{M} = 10 \cdot (r/\text{au})^{-3} \cdot \pi \left(\frac{3\big(M_\text{planet}/[g]\big)}{4\pi(\rho_\text{internal}/[g/\text{cm}^3])}\right)^{2/3} \cdot 2\pi \text{ g/year}$$

Combining all the constants gives us $20\pi^2(3/4\pi)^{2/3} = 75.96$, so call it 100.

$$\dot{M} = 100(r/\text{au})^{-3}\left(\frac{M_\text{planet}/[g]}{\rho_\text{internal}/[g/\text{cm}^3]}\right)^{2/3} \text{ g/year}$$

The timescale is

$$t = M/\dot{M} = 0.01\big(M_\text{planet}/[g]\big)^{1/3}\big(\rho_\text{internal}/[\text{g/cm}^3]\big)^{2/3}(r/\text{au})^3$$

For Earth, which has $M \sim 6 \times 10^{27}$ g (for order-of-magnitude estimation and convenience with the 1/3 I said it's just 10^{27} g) and $\rho_\text{internal} \sim 5.5 \text{ g/cm}^3$ (also for convenience I said this is 1), we get

$$t_\text{Earth} = 5.66 \times 10^7 \text{ years}$$

And for Neptune, which has $M \sim 10^{29}$ g, $\rho_\text{internal} \sim 1.6\text{g/cm}^3$ and $r \sim 30$ au, we get

$$t_\text{Neptune} = 1.71 \times 10^{12} \text{ years}$$

The age of the Universe is 13.8 billion years, or 1.38×10^{10} years, so this process provides a timescale that is too long to describe the formation of Neptune.