Astr 118, Physics of Planetary Systems
Discussion Week 7: Atmospheres

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1. Planet formation times

Last time, we found that the core accretion time for a planet around the Sun was about

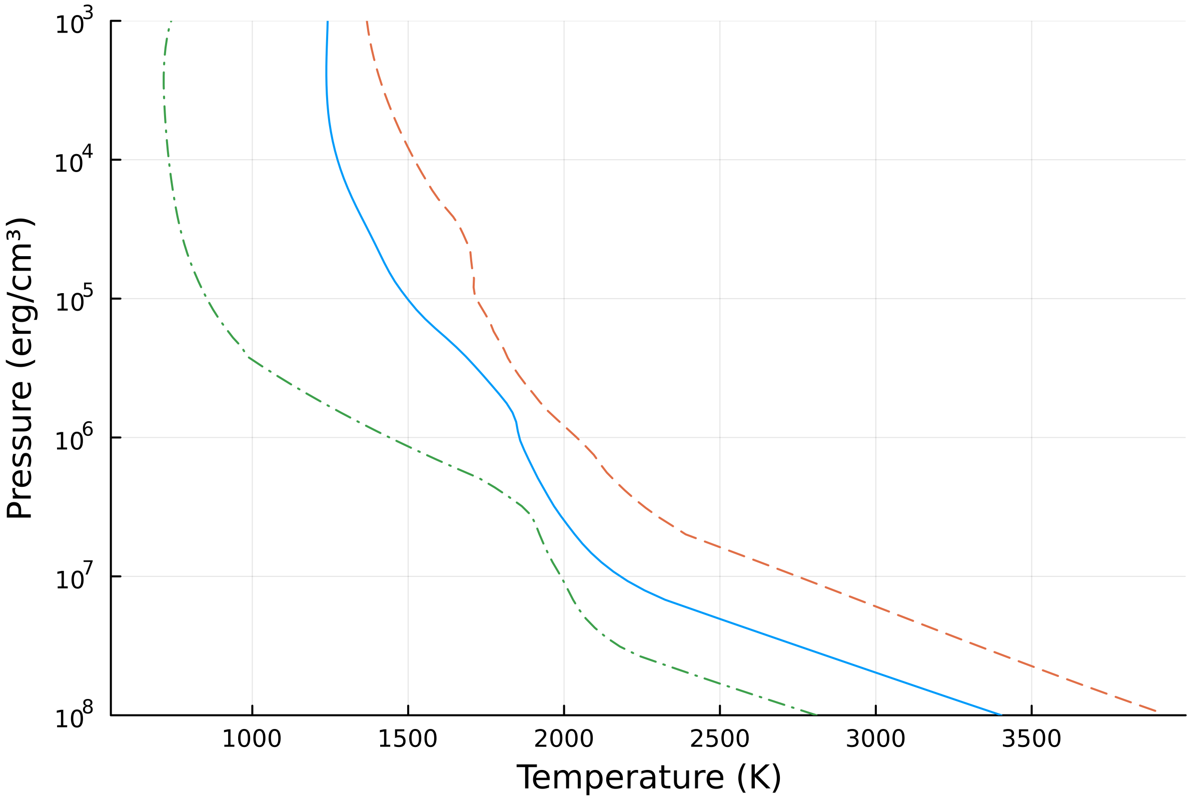
$$t_{\text{accr}} = M/\dot{M} = 0.01\left(M_{\text{planet}}/[g]\right)^{1/3}\left(\rho_{\text{internal}}/[\text{g/cm}^3]\right)^{2/3}(r/\text{au})^3.$$

a. Calculate this time for Earth ($M \sim 6 \times 10^{27}$ g, $\rho_{\text{internal}} \sim 5.5$ g/cm^3).

b. Calculate this time for Neptune ($M \sim 10^{29}$ g, $\rho_{\text{internal}} \sim 1.6$g/cm^3, $r \sim 30$ au).

2. Temperature structures and where clouds form

A basic quantity in understanding atmospheres is the *temperature structure*, which is a function $T(P)$: the temperature at each atmospheric pressure. Atmospheric pressure is monotonic, so you could equivalently write this as $T(z)$ if you had an expression for $P(z)$ (which you can get out of hydrostatic equilibrium.)



a. Since the axes can be a little strange, let's get used to reading P-T profiles! Which of these is the coldest and which is the hottest? If these planets have equilibrium temperatures of 1000K, 1500K, and 2000K, identify the *photosphere* for each: the pressure at which we're observing the planet's blackbody temperature.

b. Clouds form in planets at the lowest point where the condensate's partial pressure (pressure times abundance) exceeds a known vapor pressure. Suppose we have a condensate with an abundance of about 10^{-5} and a vapor pressure expression

$$P_{\text{vapor}}(T) = 10^{10-15000/T} \text{ erg/cm}^3.$$

Indicate on the plot the approximate points at which clouds form for each of these planets.

Solving for exact points is hard in general, and impossible here since I haven't given you the P-T profile datapoints; to get a general idea, try approximately sketching P_{vapor}/abundance over the P-T profiles.

3. The possible shapes of temperature structures

An approximate expression for the temperature structure of an atmosphere is

$$T(\tau)^4 = \frac{3}{4}T_{\text{int}}^4\left(\tau + \frac{2}{3}\right) + \frac{3}{4}T_{\text{irr}}^4\left[\frac{2}{3} + \frac{1}{\gamma\sqrt{3}} + \left(\frac{\gamma}{\sqrt{3}} - \frac{1}{\gamma\sqrt{3}}\right)\exp\left(-\gamma\tau\sqrt{3}\right)\right]$$

This is a mess, so let's try and get something sensible out of it!

- τ is "optical depth": this is about proportional to pressure – it's small high up and large lower down.
- T_{int} and T_{irr} are the internal and irradiation temperatures of the atmosphere: the amount of heating we get from the planet interior and the stellar radiation respectively.
- γ is something like "efficiency of planet heating"/ "efficiency of star heating". If it's < 1, star heating permeates the atmosphere more, and if it's > 1, planet heating permeates more.

a. In a free-floating planet, what does the temperature structure look like?

b. In a highly irradiated atmosphere where $T_{\text{irr}} \gg T_{\text{int}}$, what happens for $\tau \ll 1$ (upper atmosphere) and $\tau \gg 1$ (lower atmosphere)?

c. What's likely to be true about planets that exhibit *thermal inversions*, where the atmosphere gets colder and then warmer again as you're going up? If you get the chance, try putting this into Python (or Desmos, but I think you'd have to use $y = \log\tau$ to get interesting-looking structures on one screen) and seeing how the profile varies as you change the three free parameters.

4. Horizontal wind speed

There's a lot to cover in 1D analysis of atmospheres, but there are also some effects that need us to consider the 3D structure. Let's look at just one: zonal winds. These are described by the partial differential equation

$$\frac{2\pi}{P_{\text{rotation}}}\frac{\partial u}{\partial \ln p} = -\frac{\partial(RT)}{\partial x}$$

where the wind speed is u, the length of the planet's day is P_{rotation}, and p, T, x describe variation across the planet in the horizontal direction. Approximating $\frac{\partial y}{\partial x} \approx \frac{\Delta y}{\Delta x}$ for reasonable choices of Δy, Δx, how fast are Earth's winds? Plug this into WolframAlpha so you can avoid a bunch of tedious unit-chasing!

What might change in the exoplanet context?