# Astr 118, Physics of Planetary Systems Discussion Week 8: Atmospheres 

Aditya Sengupta, adityars@ucsc.edu, ISB 127

## 1. The possible shapes of temperature structures

An approximate expression for the temperature structure of an atmosphere is

$$
T(\tau)^{4}=\frac{3}{4} T_{\text {int }}^{4}\left(\tau+\frac{2}{3}\right)+\frac{3}{4} T_{\text {irr }}^{4}\left[\frac{2}{3}+\frac{1}{\gamma \sqrt{3}}+\left(\frac{\gamma}{\sqrt{3}}-\frac{1}{\gamma \sqrt{3}}\right) \exp (-\gamma \tau \sqrt{3})\right]
$$

This is a mess, so let's try and get something sensible out of it!

- $\tau$ is "optical depth": this is about proportional to pressure - it's small high up and large lower down. When coding, we'll just pretend this is pressure whenever it's convenient.
- $T_{\text {int }}$ and $T_{\text {irr }}$ are the internal and irradiation temperatures of the atmosphere: the amount of heating we get from the planet interior and the stellar radiation respectively.
- $\gamma$ is something like "efficiency of planet heating"/ "efficiency of star heating". If it's $<1$, star heating permeates the atmosphere more, and if it's $>1$, planet heating permeates more.
In pairs, implement and plot this in Python. Let $\tau$ range from $10^{-8}$ to $10^{3} \log$-spaced, and use $T_{\mathrm{int}}, T_{\mathrm{irr}}, \gamma$ (gamma) as free parameters. I highly recommend implementing this as a function that takes in the three free parameters and returns the $T$ array, so you can overplot many results at once. Use this visualization to answer the following questions.
a. In a free-floating planet, what does the temperature structure look like?
b. In a highly irradiated atmosphere where $T_{\text {irr }} \gg T_{\text {int }}$, what happens for $\tau \ll 1$ (upper atmosphere) and $\tau \gg 1$ (lower atmosphere)?
c. What's likely to be true about planets that exhibit thermal inversions, where the atmosphere gets colder and then warmer again as you're going up?


## 2. Horizontal wind speed

There's a lot to cover in 1D analysis of atmospheres, but there are also some effects that need us to consider the 3D structure. Let's look at just one: zonal winds. These are described by the partial differential equation

$$
\frac{2 \pi}{P_{\text {rotation }}} \frac{\partial u}{\partial \ln p}=-\frac{\partial(R T)}{\partial x}
$$

where the wind speed is $u$, the length of the planet's day is $P_{\text {rotation }}$, and $p, T, x$ describe variation across the planet in the horizontal direction. Approximating $\frac{\partial y}{\partial x} \approx \frac{\Delta y}{\Delta x}$ for reasonable choices of $\Delta y, \Delta x$, how fast are Earth's winds? Plug this into WolframAlpha ( $R=$ "specific gas constant of air") to avoid a bunch of tedious unit-chasing! What might change in the exoplanet context?

## 3. Converting to $T(z)$ with pair programming!

I have a Jupyter notebook with a P-T profile loaded. Talk me through the steps I need to do to convert the independent axis from pressure $[T(P)]$ to altitude $[T(z)]$ ! The ideal gas law $P=\rho R T / \mu$ and hydrostatic equilibrium $\mathrm{d} P / \mathrm{d} z=-\rho g$ may be useful.

