Astr 118, Physics of Planetary Systems
Discussion Week 9: Interiors

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1. Horizontal wind speed

There's a lot to cover in 1D analysis of atmospheres, but there are also some effects that need us to consider the 3D structure. Let's look at just one: zonal winds. These are described by the partial differential equation

$$\frac{2\pi}{P_{\text{rotation}}}\frac{\partial u}{\partial \ln p} = -\frac{\partial (RT)}{\partial x}$$

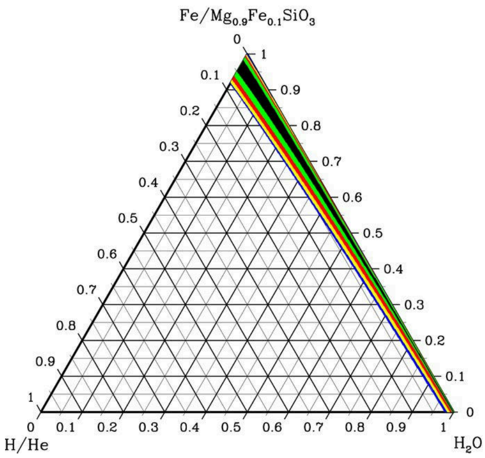
where the wind speed is u, the length of the planet's day is P_{rotation}, and p, T, x describe variation across the planet in the horizontal direction. Approximating $\frac{\partial y}{\partial x} \approx \frac{\Delta y}{\Delta x}$ for reasonable choices of Δy, Δx, how fast are Earth's winds? Plug this into WolframAlpha ($R =$ "specific gas constant of air") to avoid a bunch of tedious unit-chasing! What might change in the exoplanet context?

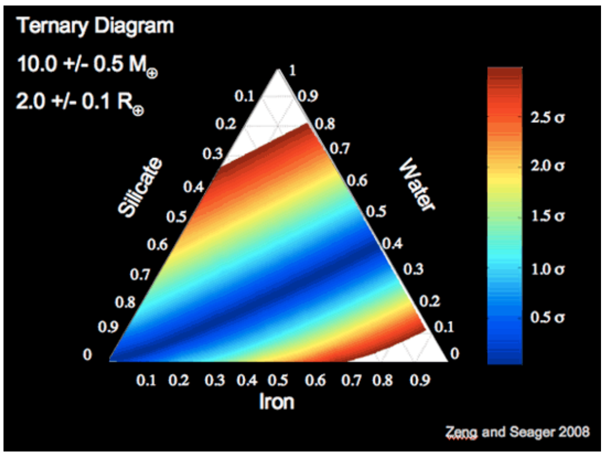
2. Reading ternary diagrams

Ternary diagrams are a way of representing three interrelated components of a planet's composition. You can trace a point back to its axes the same way you would with a rectangular plot, just along diagonal lines instead of vertical/horizontal ones. I'll sketch one of these on the board and indicate how to get the coordinates of a point.

a. For the plot on the left, what kinds of feasible compositions are there? Which elements are present and which are mostly not?

b. For the plot on the right, pick a point in the blue ($< 0.5\sigma$) region and find its percentage of silicate, water, and iron.





(Question 3 on the other side!)

3. Radius and luminosity over time

Giant planets contract and give off less light over time, and we can figure out the rate at which this happens! Suppose we have a planet of mass M, radius R, and temperature T. We'll say M and T are constants, and we want to figure out how R varies with time.

a. Write down expressions for the gravitational binding energy E and the luminosity L in terms of M, R, T.

b. Luminosity and energy are related by $L = \mathrm{d}E/\,\mathrm{d}t$. Solve this equation proportionally (you can collect constants like π, σ, G, M, T into one variable) to get a scaling relation $R \propto t^{?}$.

c. Use this result to derive a scaling relation for luminosity, $L \propto t^{?}$.

d. Estimate the relationship between luminosity and age using one of the red curves in the diagram shown here. How close did our calculation get? Why might this be the case?