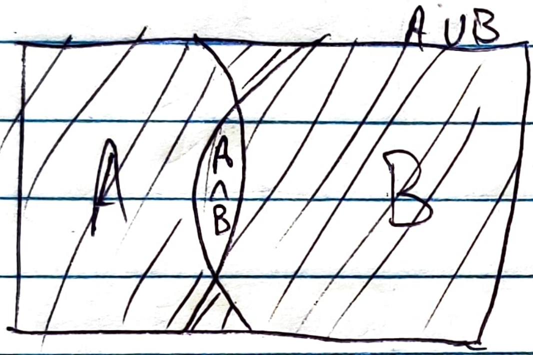
EECS 126 Discussion 1

Aditya's OH
W 2-4 PM

Starting at 3:10 PM (Berkeley time)

→ introductions + 3-4 min breakout room activity
→ quick discussion of recent changes to the class, resources
→ problems!

1. a) Probability that exactly one of the events A or B happens

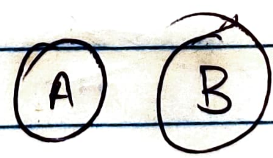


$$P(\text{A} \overset{\text{one of}}{\text{or}} \text{B}) = P(A\cup B) - P(A\cap B)$$
$$= P(A) + P(B) - 2P(A\cap B)$$

$$P(\text{only } A) = P(A) - P(A\cap B)$$
$$P(\text{only } B) = P(B) - P(A\cap B)$$

$$P(\text{only } A \text{ or only } B) = P(A) + P(B) - 2P(A\cap B)$$

if $A\cap B = \emptyset$



b)

b) A independent of itself.

$P(A \cap A) = P(A)$ solution in staff sol.

A, B indep $\Rightarrow P(A|B) = P(A)$

$P(A|A) = P(A)$

if A ever happens, $1 = P(A)$

if A never happens, $0 = P(A)$

2 A_k, $k \in \{1, 2, \dots, n\}$

"At least one of the A_ks occur".

$P(\bigcup_{k=1}^{n} A_k) = 1$

Looks like union bound.

$1 = P(\bigcup_{k=1}^{n} A_k) \leq \sum_{k=1}^{n} P(A_k) = np$

$1 \leq np \Rightarrow \boxed{p \geq 1/n}$

$P(A_i \cap A_j) = q$ for $i, j \in \{1, \dots, n\}, i \neq j$

n choices for i,

$n-1$ choices for j,

div by 2 for double count.

$P(A_i \cap A_j \cap A_k) = 0$ for $i \neq j \neq k$

$$\boxed{P\left(\bigcup_{i,j} (A_i \cap A_j)\right) \leq 1 \Rightarrow \sum_{\substack{i,j \\ i<j}} P(A_i \cap A_j) \leq 1 \qquad 1 \geq P(A_i \cap A_j) \cdot \frac{n(n-1)}{2}}$$

$P(A_i \cap A_j) \Downarrow q$

I said this was union bound, it is not - clarification next page

$$\boxed{q \leq \frac{2}{n(n-1)}}$$

3. Sampling from : the set of all inscribed cubes.

Want : there exists at least one all red cube.

$P(\text{all red vertices}) > 0$, or $P(\text{any blue vertices}) < 1$

B_1 : vertex 1 is blue

B_2 : vertex 2 is blue

$\vdots$

B_8

WTS: $P(B_1 \cup B_2 \cup \ldots \cup B_8) < 1$

$P(\bigcup B_i) \leq \sum_{i=1}^{8} P(B_i) < 8 \times \frac{1}{10} = \frac{8}{10} < 1$

$P(\bigcup B_i) \leq \sum_i P(B_i) < 1$ ✓

2. joint event case:

$$P(A_i \cap A_j \cap A_k) = 0 \quad \forall \quad i \neq j \neq k.$$

So each $A_i \cap A_j$ is disjoint.

Countable additivity tells us:

$$1 \geq P\left(\bigcup_{(i,j)\,|\,i<j} (A_i \cap A_j)\right) = \underbrace{\sum_{(i,j)\,|\,i<j}}_{\frac{n(n-1)}{2}} \underbrace{P(A_i \cap A_j)}_{q}$$

$$1 \geq \frac{n(n-1)}{2} q$$

$$\boxed{q \leq \frac{2}{n(n-1)}}$$

Probabilistic Method: use probability to solve new kinds of math problems!

Sample from a set and consider the probability of a certain outcome:

$\longrightarrow$ if $\mathbb{P}$(property) $= 0$, all of them fail to have the property

$\longrightarrow$ if $\mathbb{P}$(property) < 1, there's at least one object that fails to have the property.
(for n large enough)

Statements about probability, but conclusions are certain!