EECS 126 Discussion 8

→ HW7 self-grade due tonight

→ Today's problems:

- System Shocks - exponentials / Poisson process
- Spatial Poisson processes - fun interpretation of a PP
- M-M-∞ queues - CTMC practice

Math Tricks / Facts

$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

$\int_0^{\infty} e^{-ax^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$

CDF of $Exp(\lambda) = 1 - e^{-\lambda x}$

1. a) Failure if any $X_i > \gamma$

$P((X_1 > \gamma) \cup (X_2 > \gamma) \cup \dots \cup (X_n > \gamma))$

$= 1 - P((X_1 \le \gamma) \cap (X_2 \le \gamma) \cap \dots \cap (X_n \le \gamma))$

$= 1 - (1 - e^{-\gamma})^n$

b) Poisson process over "shock space"

Failure condition: fewer than n "shock space" arrivals in "shock time" γ.

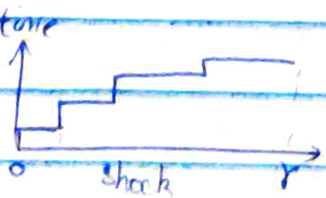
(an arrival is +1 to time, so time < n is < n arrivals)

$P(\sum_{i=1}^{n} X_i > \gamma)$

↳ Erlang distribution (k, λ) PDF: $\frac{\lambda^k x^{k-1} e^{-\lambda x}}{(k-1)!}$

Random Incidence Paradox

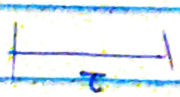
$(N_t)_{t \ge 0}$ is a $PP(1)$



$\to P(N_\gamma < n)$

$= \sum_{k=0}^{n-1} P(N_\gamma = k)$

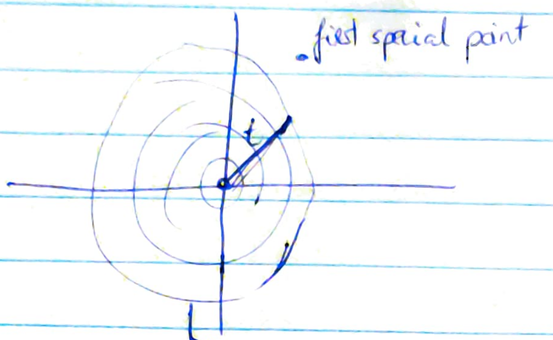
↳ $Pois(1 \cdot \gamma)$



$= \sum_{k=0}^{n-1} \frac{\gamma^k e^{-\gamma}}{k!}$

↳ $N \sim Pois(\lambda \cdot \tau)$

2. a)



$P(X \geq t) = P($ no special point here $)$

$N_{s,t} \sim \text{Pois}(\lambda \pi t^2)$

$P(X > t) = P(N_{s,t} = 0) = \cancel{\frac{1}{\lambda}} e^{-\lambda \pi t^2}$.

b) $E[X] = \int_0^\infty P(X > t)\,dt$

$$= \int_0^\infty e^{-\lambda \pi t^2}\,dt = \int_0^\infty e^{-\frac{t^2}{2\sigma^2}}\,dt$$

$$\sigma = \frac{1}{\sqrt{2\pi\lambda}}$$

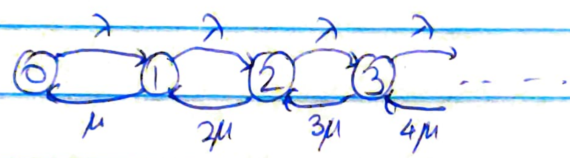
$$E[X] = \sigma\sqrt{2\pi} \cdot \frac{1}{2} = \frac{1}{2\sqrt{\lambda}}.$$

3. M-M-∞ Queues

a) The process is memoryless.

- Customers arrive independently
- Customers are served independently
- Service times are memoryless.

MC states? $k = 0, 1, 2, \ldots$



b) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$

$$\pi(k) \cdot Q(k, k+1) = \pi(k+1)\, Q(k+1, k)$$

$$\pi(k) \cdot \lambda = \pi(k+1) \cdot (k+1)\mu$$

Take $\pi(0)$ to be solved for

$$\pi(1) = \pi(0) \cdot \frac{\lambda}{\mu}$$

$$\pi(2) = \pi(1) \cdot \frac{\lambda}{2\mu} = \pi(0) \cdot \frac{\lambda^2}{2\mu^2}$$

$$\pi(k) = \pi(0) \cdot \left(\frac{\lambda}{\mu}\right)^k \cdot \frac{1}{k!}$$

$$1 = \pi(0) \sum_{k=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^k \frac{1}{k!} \Rightarrow \pi(0) = e^{-\lambda/\mu}$$

Positive recurrence!