Radiative Processes Reconstruction

Aditya Sengupta

June 2023

1 Overall radiation theory

We're interested in the physics relating energy, mass density, number density, energy density, pressure, and temperature. For a system of particles with mass m, the pressure and energy density are related according to $P = \frac{2}{3}U$ for non-relativistic particles and $P = \frac{1}{3}U$ for relativistic particles. We'd like to relate either of these to density or temperature via an equation of state.

If we have thermal equilibrium driven by macroscopic physical interactions, we need the mean free path $l = (n\sigma)^{-1}$ to be less than some length scale of the system and the mean collision time $t = (n\sigma v)^{-1}$ to be less than some time scale of the system. In these terms, we can describe sizes in terms of optical depth, $\mathrm{d}\tau = n\sigma \, \mathrm{d}z = \alpha \, \mathrm{d}z = \kappa\rho \, \mathrm{d}z$.

All of these are set by the characteristic energy scales of physical phenomena; these include

- **ionization**: happens if $kT > \epsilon_a$. We can compare this just in temperatures using $T = 10^4$ K $\to \epsilon = 1$ eV. Half ionization is at about 1/10 of this energy.
- **degeneracy**: happens if $kT \ll \epsilon_F$. Electrons are tightly packed together and the pressure and density are related by $P \propto n^{5/3}$ (non-relativistic) and $P \propto n^{4/3}$ (relativistic).
- **neutral molecular collisions**: process with cross-section of about $\sigma \sim 10^{-16}\text{cm}^2$. $\tau_{pp} = 43\tau_{ee}$.
- **electron-electron collisions**: process with length scale of $b = \frac{2Zq^2}{m_e v^2}$.
- **electron-ion collisions**: process dominated by v_p instead of v_e, so slower than e-e. $\tau_{pe} = 1836\tau_{ee}$.
- other interactions that can provide a smaller mean free path, like magnetic effects.

2 Characteristic wavelengths of radiative processes

2.1 Radio (3 cm to 10 m)

The dominant physical process here is molecular rotation.

- synchrotron radiation from ISM electrons provides a background
- H II emission of thermal bremsstrahlung
- radio galaxies
- neutral hydrogen at the 21 cm line, possibly redshifted

2.2 Microwave and sub-millimeter (0.02 cm to 3 cm)

The dominant physical process here is molecular rotation.

- dust clouds, hydrogen gas, quasars
- cosmic microwave background

2.3 Infrared (800 μm to 0.01 cm)

The dominant physical processes here are atomic transitions and molecular vibrations. There's strong extinction from Earth's atmosphere (carbon dioxide, water) and interstellar/interplanetary dust.

- dust heated to 400-4000 K
- redshifted light from galaxy formation

2.4 Optical and UV (10 μm to 800 μm)

The dominant physical processes here are atomic transitions, molecular transitions, and molecular vibrations.

- stars, galaxies, quasars
- background galactic light

2.5 X-ray and gamma (0.0003 μm to 10 μm)

The dominant physical processes here are atomic transitions, and nuclear transitions for gamma rays. Strong absorption from Earth's atmosphere

- quasars, hot ionized gas in galaxy clusters
- gamma: accretion of matter on compact objects (can't create thermally)

3 Kinetic theory and radiative transfer

We describe radiation in terms of specific intensity $I_\nu(\boldsymbol{r}, \boldsymbol{n}, t) = \frac{2h\nu^3}{c^3} f(\nu, \boldsymbol{r}, \boldsymbol{n}, t)$. Intensity is energy per everything (time, frequency, detector area, solid angle on the sky).

$I_\nu \,/\, \nu^3$ is a conserved quantity for photons in free space. This lets us estimate surface brightness in cases where the wavelength shifts. For example, with a redshift z, ν drops by a factor $1 + z$, so I_ν drops by a factor $(1 + z)^3$, and the surface brightness $(\int I_\nu \, \mathrm{d}\nu)$ drops by a factor $(1 + z)^4$.

How do physical processes give rise to specific intensities, and how does that give rise to observable signatures? The latter is governed by the *radiative transfer equation*,

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = \kappa_\nu \rho (S_\nu - I_\nu)$$

assuming steady state. Here, S_ν is the *source function*; in the case of thermal radiation, $S_\nu = B_\nu$. The solution to the radiative transfer equation is

$$I_\nu(\tau_\nu) = I_\nu(0) e^{-\tau_\nu} + S_\nu (1 - e^{-\tau_\nu}).$$

The greater τ_ν gets, the more significant the source function becomes, and the less of the intensity behind it we can see. In general, the source function is $S_\nu = j_\nu \,/\, \alpha_\nu$: the emission divided by the absorption.

4 Combining radiative sources

If we have many processes contributing to the source function, we combine their opacities according to:

- in a fixed channel of energy transfer (or at a given energy), the highest opacity dominates and the opacities add linearly.
- across channels of energy transfer (or over a spectrum), the lowest opacity dominates and the opacities add inversely.

These channels include photon interactions with free electrons, atoms, molecules, and dust.

1. With free electrons, we have electron scattering and free-free absorption (inverse bremsstrahlung).
2. With atoms, we have bound-bound absorption (transitions between atomic levels) and bound-free absorption (ionization).
3. Molecules and dust don't usually dominate.

In high optical-depth cases, we can take a mean opacity across frequencies and combine them harmonically, weighted with the blackbody function, to get the *Rosseland mean opacity*:

$$\frac{1}{\kappa} = \left[\int_0^\infty \frac{1}{\kappa_\nu}\frac{\partial B_\nu}{\partial T}\,\mathrm{d}\nu\right]\left[\int_0^\infty \frac{\partial B_\nu}{\partial T}\,\mathrm{d}\nu\right]^{-1}$$

5 Einstein coefficients

In cases where we don't have LTE, like in the ISM, we need to look at the rates at which various processes happen and balance them appropriately. For atomic transitions, these rates are the *Einstein coefficients*. If we have a two-level system with densities n_1, n_2 in states 1, 2, we balance three processes:

- photon absorption causing transitions $1 \to 2$, governed by B_{12}
- stimulated emission causing transitions $2 \to 1$, governed by B_{21}
- spontaneous emission causing transitions $2 \to 1$ without any relationship to incident radiation, governed by A_{21}.

We can balance this to get

$$n_1 B_{12}\overline{J} = n_2 A_{21} + n_2 B_{21}\overline{J}.$$

Further, we have

$$\frac{n_1}{n_2} = \frac{g_1}{g_2}\exp\left(\frac{h\nu_0}{kT}\right)$$

and substituting this into the balance equation and rearranging gives us

$$\overline{J} = \frac{A_{21} \;/\; B_{21}}{\frac{g_1 B_{12}}{g_2 B_{21}}\exp\left(\frac{h\nu_0}{kT}\right) - 1}.$$

In thermal equilibrium, setting this equal to $B_\nu(T)$ gives us the Einstein relations $g_1 B_{12} = g_2 B_{21}$ and $\frac{A_{21}}{B_{21}} = \frac{2h\nu^3}{c^2}$. Since the coefficients can't depend on the whole spectrum, these always hold even outside of thermal equilibrium.

In order to get something useful for radiative transfer, we need emission and absorption functions so that we can get a source function. The emission is

$$j_\nu = \frac{h\nu_0 A_{21} n_2 \phi(\nu)}{4\pi}$$

and the absorption is

$$\alpha_\nu = \frac{h\nu(n_1 B_{12} - n_2 B_{21})\phi(\nu)}{4\pi}.$$

We can also compute the cross-section using the Einstein relations and the fact that $\alpha_\nu = n_1\sigma_{12}$:

$$\sigma_{12} = \frac{\lambda^2}{8\pi}\frac{A_{21}}{\Delta\nu}$$

6 Scattering and diffusion

Scattering can be described as a diffusive random-walk process, in which a particle takes a number of steps in random directions with the same step size. This behavior can be described well by the diffusion equation, $\frac{\partial n}{\partial t} = D\nabla^2 n$. D is the *diffusion constant*, and it sets the speed/length scale over which diffusion happens. $D = vl = \frac{l^2}{\tau}$ helps us find what this is, if we know the mean free path and the velocity of scattering particles, and we can use this to find the population of diffusing particles over time and space.

7 Bremsstrahlung

Bremsstrahlung, or "braking radiation", is a form of free-free emission caused by electron-proton interactions that accelerate the electron. Thermal bremsstrahlung, i.e. bremsstrahlung caused by interactions between electrons and protons at thermal velocities, has an emissivity given by

$$j_\omega = \left(\frac{q^6}{m_e^2 c^3}\right)\left(\frac{m_e}{kT}\right)^{1/2} n_e n_i \propto n^2 T^{-1/2} \quad \text{for } \hbar\omega \leq kT.$$

and therefore an integrated emissivity of $j \propto n^2 T^{1/2}$. The bremsstrahlung spectrum (in νF_ν vs ν) is flat at about the value of blackbody peak for much smaller frequencies than blackbody radiation, and drops off roughly where the blackbody peak drops off.

We're only looking at thermal bremsstrahlung, so the absorption is given by $\alpha_\nu = j_\nu \;/\; B_\nu(T)$.

8 Opacities

Different physical processes have different characteristic opacities/scaling laws for those opacities, and different shapes that are created in spectra.

For free-free absorption, putting it in equilibrium with thermal bremsstrahlung gives us $\sigma_{ff} \propto n\nu^{-2}T^{-3/2}$, and an opacity of $\kappa_{ff} \propto \rho T^{-7/2}$. This creates the overall shape of thermal bremsstrahlung on a spectrum.

For bound-free absorption, we put the rates of photoionization and recombination in equilibrium. The recombination rate is proportional to $\rho^2 T^{1/2}$ because it's directly proportional to the electron density, the ion density, the encounter velocity v, the process cross-section $\pi\lambda^2 = \pi\left(\frac{h}{m_e v}\right)^2$, and the energy released per interaction $\frac{1}{2}m_e v^2$. The photoionization rate is proportional to $F_{\text{rad}}\rho_{\text{atom}}\kappa_{bf}$, and since $F_{\text{rad}} \propto T^4$, putting these in balance also gives us $\kappa_{bf} \propto \rho T^{-7/2}$. However, since the effect of this process is to kick electrons out, which has a minimum energy but no maximum energy, we see edge features on a spectrum as a result: sharp cutoffs at the minimal energy, followed by a steady slope back to the continuum set by the blackbody or thermal bremsstrahlung. For $h\nu \sim kT$, the exponent can get closer to 3. Below the ionization threshold $\nu_I = \epsilon_a \;/\; h$, the cross-section is 0. For heavier elements, we have $\sigma_{bf} \propto Z^4\nu^{-3}$.

Bound-bound opacities are only strong at the precise frequencies of the corresponding transitions.

Which opacity dominates is physically set by the ionization fraction, which we can find using the Saha equation. For our purposes, that is

$$\frac{y^2}{1-y} = \frac{4 \times 10^{-9}}{\rho} T^{3/2} \exp\left(-1.6 \times \frac{10^5}{T}\right)$$

where $y = n_+ \;/\; n = n_e \;/\; n$.